

Moral Hazard in Lending and Labor Market Volatility*

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December 29, 2017

Abstract

When the economy experiences a sharp economic downturn, credit spreads widen and project financing costs for firms rise as funding sources begin to dry up. The economy experiences a lengthy recovery, with unemployment rates slow to return to “full employment” levels. We develop a model that displays these features. It relies on an interaction between labor search frictions and firm-level moral hazard that is accentuated during recessions. The model is capable of addressing the “Shimer puzzle,” with labor market variables exhibiting significantly more volatility on average as a result of the heightened moral hazard concerns during these episodes that significantly deepen and prolong periods of high unemployment, as vacancy postings fall dramatically and the job-finding rate declines. Our mechanism is also found to induce internal shock propagation causing the peak response of output, unemployment, and wages to occur with a several quarter delay relative to a model without such frictions. Many other labor market variables also show slower recovery—their return to pre-shock level occurs at a slower pace for a number of periods after the peak response.

Keywords: Moral Hazard, Occasionally Binding Constraints, Financial Frictions, Search Frictions, Generalized Stochastic Simulation Algorithm.

*We thank Karthik Athreya, Tor Einarsson, Huberto Ennis, Thomas Lubik, Ned Prescott, Felipe Schwartzman for their comments, suggestions, and support. We also thank seminar participants at the Federal Reserve Bank of Richmond where an earlier version of this paper was presented under the title “Labor Search with Financial Frictions”. This paper also benefited from value feedback from the participants of the CEF 2013 conference and the Fall 2013 Midwest Macroeconomic Meetings.

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1 Introduction

Shimer (2005) and Hall (2005) show that the standard Diamond-Mortensen-Pissarides (DMP) labor market search model (see Diamond 1982; Pissarides, 1985; Mortensen and Pissarides, 1994; and Pissarides, 2000) cannot replicate the high relative volatility found in the U.S. labor market data, due to productivity shocks being largely absorbed by wages. This “Shimer puzzle” has received a great deal of attention in the literature within the context of the standard DMP model that incorporates risk-neutral agents into an economy devoid of capital, i.e. where the production technology is linear in labor. Success in raising labor market volatility has been made by Shimer (2005), Hall (2005), Hall and Milgrom (2008), and Pissarides (2009), who suggest that imposing sticky wages contracts can force the productivity shocks to be increasingly absorbed by vacancies. Hagedorn and Manovskii (HM) (2008) offer an alternative explanation that simply requires a recalibration of the standard DMP model. In their calibration, they target the small vacancy posting costs observed in the data, and shift the wage bargaining power from a value that satisfies the Hosios (1990) criterion to one that strongly favors the firm. By increasing (decreasing) firm’s (worker’s) bargaining power, they are able to target the low empirical elasticity of wages with respect to average labor productivity. The combination of small vacancy posting costs and moderately procyclical wages yields small accounting profits that are highly volatile over the business cycle. As a result, firms have an increased incentive to post additional vacancies during expansions and reduce vacancy creation during downturns. While this calibration was successful in resolving the Shimer puzzle in the standard DMP model, Atolia, Gibson, and Marquis (2016b) demonstrate the difficulty in generating sufficient labor market volatility within a basic real business cycle (RBC) model with DMP search frictions using the HM calibration.¹ Therefore, additional mechanisms are needed to further amplify labor market volatility within this more general model.

In seeking additional sources of volatility, this paper joins others in pursuing the considerable evidence that credit market frictions induce significant amplification of cyclical fluctuations in the economy.² Petrosky-Nadeau and Wasmer (2013) (P-NW) introduce credit

¹Moving from the standard DMP model to an RBC search model typically requires one to drop the assumption of risk-neutrality and an infinite Frisch elasticity of labor supply. In a related paper, Jung and Kuester (2011) demonstrate that the volatility of labor market variables declines as the degree of curvature in the utility function is increased.

²Considerable empirical evidence exists which suggests that credit constraints not only amplify output fluctuations (as in Williamson 1986, 1987; Bernanke and Gertler 1989; Kiyotaki and Moore, 1997; and Bernanke, Gertler, and Gilchrist, 1999) but also lead to greater volatility in the labor market. For example, Sharpe (1994) uses micro-level data to establish a relationship between firm leverage and the cyclicity of

market frictions in the standard DMP model (with risk-neutral agents) and demonstrate that additional costs incurred by both borrowers and lenders associated with creating a credit match are sufficient to generate additional volatility in the labor market inducing estimates of the elasticity of labor market tightness with respect to productivity that are in line with U.S. data.³ In a subsequent paper, Petrosky-Nadeau (P-N) (2014) generates endogenous persistence in labor market variables by introducing a variety of credit market frictions, including agency costs associated with costly state verification (as in Carlstrom and Fuerst, 1997; Townsend, 1979; Gale and Helwig, 1985; and Williamson, 1987) that is counter-cyclical due to fluctuations in net worth.⁴ While their financial frictions amplify labor market volatility in a model with log-preferences, additional sources of volatility (separate credit shocks and counter-cyclical recovery costs) are required to resolve the Shimer puzzle.

This paper introduces financial frictions into a basic RBC model with DMP labor search frictions (see Merz, 1995 and Andolfatto, 1996 for a discussion of RBC search models). The financial frictions are of a different sort than those explored in P-NW and P-N. They enter the model through firm-level moral hazard that exists between the entrepreneurs and their investors due to the entrepreneurs' ability to alter their funded project's probability of success through their choice of effort (high/low). This approach to modelling financial frictions is described in Holmstrom and Tirole (1997, 1998), Tirole (2006) and Atolia, Einarsson,

employment, ultimately finding that more highly levered firms have a more highly cyclical and volatile labor demand. Caggese and Cunat (2008) find a result similar to Sharpe (1994), but they focus on financially constrained firms' desire to avoid firing costs through the use of flexible labor contracts. Benmelch, Bergman, and Seru (2011) demonstrate that fluctuations in the availability of credit play important roles in determining both firm-level employment decisions as well as aggregate unemployment outcomes. At the firm level, they demonstrate that fluctuations in a firm's cash flow affects their employment levels even after controlling for contemporaneous changes in capital investment. Thus, fluctuations in cash flow exercise a direct effect on employment decisions and are not just transmitted indirectly through changes in the stock of physical capital. This finding is also consistent with those of Nickell and Nicolitsas (1999) which state that liquidity constraints directly affect the firms' employment decisions. In a more recent work, Chodorow-Reich (2014) demonstrate that firms' employment outcomes are directly influenced by their lenders' financial health. As for economy-wide responses, they find that an adverse exogenous shock to the supply of credit in a U.S. metropolitan statistical area results in a statistically significant increase in the area's unemployment rate. Monacelli, Quadrini, and Trigari (MQT) (2011) also provide evidence that credit shocks induce significant, persistent responses of employment. However, they argue that firms take on higher debt levels to gain advantage in wage bargaining and this decision leads to slower recovery of employment and wages following large adverse credit shocks.

³This additional volatility is achieved through a modification of the small surplus condition presented in HM.

⁴Empirical evidence indicates that labor market variables gradually respond to shocks, reaching their maximum amplitude with a five to six quarter delay (see Fujita and Ramey, 2007 and P-N for details). The financial frictions included in P-N are sufficient to delay the response in key labor market variables by one quarter.

and Marquis (2010), and is shown by Atolia, Gibson, and Marquis (2016a) to produce realistically asymmetric business cycles within a calibrated DSGE model. As in Atolia, Gibson, and Marquis (2016a), investors finance two-period projects in an equity market, where the funding is used by the entrepreneurs to rent capital and pay workers. However, entrepreneurs now face a vacancy posting decision where vacancy posting costs must also be covered using the funds raised in the equity market. The presence of moral hazard implies that the entrepreneurs' incentive to shirk (choose low effort) is counter-cyclical. When the economy enters a sufficiently severe recession, investors incentivize entrepreneurs to be diligent (to choose high effort) by purchasing fewer shares in their projects, thereby leaving the entrepreneurs a larger share of the profit. While this act of purchasing fewer shares successfully incentivizes the entrepreneurs, it also results in a swift and severe reduction in lending. This reduction in lending causes a sharp increase in the shadow price of loanable funds which directly influences the entrepreneur's vacancy posting decisions. This interaction between financial and labor market frictions during sharp economic downturns is seen to significantly deepen and prolong periods of high unemployment, as vacancy postings and the job-finding rate fall dramatically.⁵

These periods of high and persistent unemployment are marked by wide credit spreads in the economy. In our model, they induce sharp increases in the entrepreneur's cost of funds due to a binding incentive compatibility constraint that is required to correctly incentivize the entrepreneur not to shirk. We calibrate the agency costs in the model to match the frequency of spikes in the spread between the rate paid on 3-month non-financial commercial paper and 3-month Treasury bills observed in the data. Under this calibration, the addition of these financial frictions during periods of severe economic downturns are seen to be sufficient to bring the volatility in labor market variables (unemployment, vacancies, and labor market tightness) to values close to those observed in the data, thus offering one resolution to the Shimer puzzle.⁶ The contribution of moral hazard in lending in this model is quantified by recalibrating the model with the agency costs minimized, effectively eliminating the friction, in which case the Shimer puzzle reemerges. While resolving the Shimer puzzle within an RBC search model is our primary objective, we also find that our financial fiction improves the dynamics of labor market variables, especially unemployment. Specifically, em-

⁵Wasmer and Weil (2004) also model idea-rich, cash-poor entrepreneurs who must borrow funds to post vacancies. The financial friction they employ is a search friction that is modeled symmetrically with the DMP labor search, whereby a match must be achieved between borrowers (entrepreneurs) and lenders (financial intermediaries) for the funding to take place.

⁶The model can be extended to also match the amplitude of spikes in the spread without compromising its ability to resolve the Shimer puzzle. (See Appendix A.)

empirical evidence indicates that labor market variables such as unemployment, vacancies, and their ratio, labor market tightness, display significant endogenous persistence and respond gradually to shocks, achieving their maximum amplitude with a five to six quarter delay (See Fujita and Ramey, 2007 and P-N). While these dynamic features are not present in the standard DMP search model, we find that when financial frictions are operational, endogenous persistence is added to labor market variables and unemployment displays a hump-shaped response that is qualitatively similar to that found in the data.

The rest of the paper is organized as follows. Section 2 presents the basic structure of the model, highlighting the components of the labor market and the risky investment projects. This section also sets up the entrepreneur's and investor's problems that are central to the model. Section 3 focuses on solving the model, while section 4 discusses the model's calibration. The model's results are presented in section 5, and section 6 provides the conclusion along with a brief discussion of future research.

2 The Model

The economy is populated by a continuum of identical households of measure one. Each household consists of an investor, a continuum of workers and a continuum of entrepreneurs who pool resources and (perfectly) share risk. Entrepreneurs sell equity in their projects to outside investors, and they use the proceeds to hire labor, rent capital for use in the project, and post vacancies that are used to attract new hires. Firm-level moral hazard is present in that entrepreneurs can affect their project's probability of success through their choice of effort. If they choose to shirk their responsibilities and engage in a privately beneficial activity, their project's probability of success will fall relative to that if they were diligent. The incentive constraints that underlie the equity contracts bind only in periods of sufficiently low aggregate economic activity. The volatility of labor market variables is shown to be significantly amplified by the presence of financial frictions, bringing the model closer in line with the data.

2.1 Labor Market

The labor market structure used in this paper is a modified version of the standard DMP labor market search model, where individuals are now insulated from unemployment risk through their membership in a representative household (See Shimer, 2010). The key point of departure between our model and that found in the literature on labor market search frictions

is the presence of firm-level moral hazard and resource financing constraints. The interaction of these additions with the labor search frictions will be discussed in detail throughout the next few sections.

The timing of labor market events is as follows. At the start of any given period t a fraction of workers, n_t , are employed while $1 - n_t$ are unemployed. The employed workers supply labor inelastically to entrepreneurs while unemployed workers search for jobs. The amount of job openings available within the period is given by the number of vacancies posted by the entrepreneurs, v_t . The ratio of vacancies to unemployment, which is typically used in the literature to provide a measure of labor market tightness, is denoted by:

$$\Phi_t = \frac{v_t}{1 - n_t} \quad (1)$$

Following Hall (2005) and Hagedorn and Manovskii (2008), we assume that a constant fraction, x , of current employees will separate during the period.⁷ The number of new job matches formed within a period is governed by the following constant returns to scale function:

$$m_t = \frac{(1 - n_t)v_t}{[(1 - n_t)^\gamma + v_t^\gamma]^{\frac{1}{\gamma}}} \quad (2)$$

Having specified the matching function, we define the job-finding rate, f_t , and the vacancy-filling rate, q_t as:

$$f_t = \frac{m_t}{1 - n_t} \quad (3)$$

$$q_t = \frac{m_t}{v_t} \quad (4)$$

Given the labor market structure presented above, the aggregate evolution equation for labor in the economy is given by:

$$n_{t+1} = (1 - x)n_t + m_t \quad (5)$$

⁷This is also consistent with Shimer (2012) who find that the job finding rate is more important than the separation rate in explaining fluctuations in the U.S. unemployment rate. However, as pointed out by a referee, this assumption may not be completely innocuous in this context, since some evidence exists (Petrongolo and Pissarides, 2008) that separation rates increase during severe economic downturns, and those would coincide with times when the ICC constraint in our model binds. However, net job creation is still dominated by new hires, even during recessions, and moreover, if the separation rate rose during these sharp downturns, as firms attempted to shore up their finances by reducing their wage bill through layoffs and firings, then the impact of the financial friction on labor market volatility in this model should be exacerbated. Nonetheless, it remains for future research to determine the significance of cyclical separations during times of financial distress for firms.

The representative household and each entrepreneur takes their labor market probabilities as given. Therefore, the labor evolution equations faced by the household (denoted by s) and a single entrepreneur (denoted by i) are given respectively by:

$$n_{t+1}^s = (1-x)n_t^s + (1-n_t^s)f_t \quad (6)$$

$$n_{t+1}^i = (1-x)n_t^i + v_t^i q_t \quad (7)$$

2.2 The Entrepreneur's Problem

At the beginning of each period, the entrepreneurs of the representative household each start a new investment project indexed by $i \in [0, 1]$. These projects take two periods to complete. During the first period, date t , the existing labor must be paid and capital must be acquired for use in the project. Entrepreneurs must also post vacancies at the start of each period in order to augment the amount of labor that will be available at the start of the next period. In order to finance these resource costs, each entrepreneur sells shares, s_t^i , in their project to investors of other households at the price p_t^i .⁸ With the total shares of project i normalized to 1, the shares sold promise investors a return equal to that fraction of the project's potential revenue. The first-period financing constraint faced by entrepreneur i at date t is given by:

$$w_t n_t^i + r_t k_{t+1}^i + v_t^i G \leq p_t^i s_t^i \quad (8)$$

where n_t^i , k_{t+1}^i , and v_t^i denote labor, capital, and vacancies, while w_t , r_t , and G denote wages, the rental rate on capital, and the cost of posting vacancies respectively. Note the difference in timing between the capital rental rate, r_t , and the rented capital, k_{t+1}^i . In the model, production does not occur until the second period, $t+1$, but all input costs are paid up front at date t . The timing convention is thus chosen to date the rented capital stock with the period in which it is used in production.

At the start of the second period, date $t+1$, there is an aggregate, economy-wide productivity shock that affects all projects symmetrically. The resulting level of productivity, θ_{t+1} , along with the previously installed quantities of capital and labor determine the potential output of project i at time $t+1$, denoted y_{t+1}^i , with production technology:

$$y_{t+1}^i = \theta_{t+1} (k_{t+1}^i)^\alpha (n_t^i)^{1-\alpha}, \quad \alpha \in (0, 1) \quad (9)$$

⁸Appendix A extends the model, allowing a predetermined fraction of the resource costs to be paid using internal finance (i.e., through shares issued within the household).

Each entrepreneur sells y_{t+1}^i at a price ξ_{t+1}^i in order to generate revenue equal to:

$$\hat{R}_{t+1}^i = \xi_{t+1}^i y_{t+1}^i \quad (10)$$

These projects are inherently risky, with the output from unsuccessful projects equal to zero. Furthermore, these projects are subject to moral hazard in that an entrepreneur can affect his project's probability of success through his choice of effort. That is, the entrepreneur can lower his project's probability of success from p_H to $p_L < p_H$ by choosing to shirk his responsibilities and engage in a private activity for which he receives the private benefit B . When an entrepreneur shirks, he imposes a social cost equal to $(p_H - p_L)\hat{R}_{t+1}^i = \Delta p \hat{R}_{t+1}^i$. We assume that this cost of shirking is so great that investors choose to eliminate the possibility of shirking through provisions in the equity contracts.

Specifically, incentive compatibility constraints (ICC) of the form:

$$p_H(1 - s_t^i)\hat{R}_{t+1}^i \geq p_L(1 - s_t^i)\hat{R}_{t+1}^i + Bs_t^i \quad (11)$$

will be present to assure that entrepreneurs are always diligent, and that a project's probability of success is always p_H . Following Tirole (2006) we will rewrite the ICC more compactly as:

$$p_H(1 - s_t^i)\hat{R}_{t+1}^i \geq As_t^i \quad (12)$$

where $A = p_H \frac{B}{\Delta p}$ denotes the entrepreneur's agency rent and the right-hand side of equation (12) is the minimum payment to the entrepreneur that would preserve his incentive to not shirk. The fact that the minimum incentive compatible payment is a function of s_t^i is consistent with the standard assumption that an individual's propensity to shirk is decreasing in their stake in the project (see Holmstrom and Tirole, 1997, 1998; Tirole, 2006; Atolia, Einarson, and Marquis, 2011; and Atolia, Gibson, and Marquis, 2016a).

Due to the presence of search frictions in the labor market, as described below, the current level of employment, n_t^i , is a state variable that is taken as given by the entrepreneur. While the entrepreneur is pre-committed to this level of employment, he possesses the ability to alter the level of employment that will be available to him tomorrow by posting vacancies today. This means that the entrepreneur faces a dynamic labor choice problem. With all the inputs of the project purchased in advance, any revenue generated by the project is profit that must be divided between the shareholders. The outside investors are entitled to share s_t^i of this profit, leaving fraction $1 - s_t^i$ for the entrepreneurs. Using the prime ($'$) notation to denote next period's values, the entrepreneur's problem is to issue shares, s^i , rent capital,

k^i , and post vacancies, v^i , in the current period, in order to maximize his share, $(1 - s^i)$, of the expected future profits,

$$J(n^i) = \max_{k^i, v^i, s^i} \left[\beta E \left\{ \left(\frac{U_c'}{U_c} \right) p_H (1 - s^i) \hat{R}^i \right\} + \beta E \left\{ \left(\frac{U_c'}{U_c} \right) J(n^i) \right\} \right] \quad (13)$$

discounted using the household's stochastic discount factor, $\beta \left(\frac{U_c'}{U_c} \right)$, where β is the household's discount factor and $\left(\frac{U_c'}{U_c} \right)$ is its inter temporal marginal rate of substitution (MRS) in consumption (with U_c denoting the partial derivative of the household's utility function with respect to consumption, c). This maximization problem is constrained by the entrepreneur's labor evolution equation, equation (7), resource financing constraint, equation (8) and incentive compatibility constraints, equation (12).

2.3 Household Sector - Consumption and Investment

The representative household derives utility from consumption and disutility from work where all consumption goods are viewed as perfect substitutes. Following convention, it is assumed that all agents of the household separate at the start of the period. This means that the household's investor will only invest in projects run by the entrepreneurs of the other households. Similarly, the household's workers will only search for work in and supply labor to the projects that are managed by the other households. It is also assumed that while all of the household's agents engage in separate income generating activities throughout the period, they reassemble at the end of the period to pool their resources and consume together.⁹

The household's budget constraint is as follows:

$$c_t + \int_0^1 p_t^j s_t^j dj + k_{t+1} - (1 - \delta)k_t \leq w_t n_t^s + \int_0^1 \Pi_t^i di + r_t k_{t+1} + \int_0^1 p_H \hat{R}_t^j s_{t-1}^j dj \quad (14)$$

The left-hand side of equation (14) describes the investor's portfolio decision problem. He optimally allocates the household's resources between current consumption, investing in projects that are managed by the other households, and investing in capital. The right-hand side of equation (14) describes the sources of the household's resources. The employed workers of the household generate wage income by supplying labor services, n_t^s , to the entrepreneurs of the other households at the bargained wage rate, w_t . Conditional on success-

⁹Within the household's problem, the super-script j denotes variables that are managed by other households, while the super-script i denotes variables managed within the household.

fully completing their project, each of the household's entrepreneurs will earn profit Π_t^i on the projects they started at time $t - 1$. As for investment income, \hat{R}_t^j denotes the current return from projects that were invested in last period, while $r_t k_{t+1}$ denotes the current return on capital investment.

The household's utility function is given by:

$$U(c_t, n_t^s) = \ln c_t - \eta n_t^s \quad (15)$$

where we follow Shimer (2010) in imposing the linearity in n_t^s . This assumption is standard in models with labor market search frictions, and results in a constant marginal disutility of work, η . The household's state vector consists of the number of employed workers at the start of the period, n_t^s , the household's beginning of period capital holdings, k_t , shares held in projects started by other households last period, s_{t-1}^j , and the current level of productivity, θ_t . Using this state vector, along with the utility function defined in equation (15), the household's problem can be written as the following dynamic program:

$$V(n^s, k, s_{-1}^j; \theta) = \max_{c, k', s^j} \left[\log c - \eta n^s + \beta E \left\{ V(n^{s'}, k', s^j; \theta') \right\} \right] \quad (16)$$

where the maximization is subject to equations (6) and (14).

3 Solving the Model

Following Atolia, Gibson, and Marquis (2016a), we solve the entrepreneur's problem separate from the remainder of the household in order to isolate the effect of moral hazard on the model economy.

3.1 Solution to the Household's Problem

As is standard in models with basic labor market search friction, the workers' labor supply is predetermined. Therefore, the household's problem is simply a set of consumption/savings decisions that are made by the investor. Specifically, the investor maximizes the household's utility by allocating resources between current consumption and two competing investments,

capital and equities. The optimality conditions for these choices yield the following equations:

$$1 - r = \beta E \left\{ \left(\frac{c}{c'} \right) (1 - \delta) \right\} \quad (17)$$

$$p = \beta E \left\{ \left(\frac{c}{c'} \right) p_H \theta' k'^{\alpha} n^{1-\alpha} \right\} \quad (18)$$

Another important equation that is derived when solving the household's problem is the envelope condition with respect to current labor.

$$V_{n^s} = \frac{w}{c} - \eta + \beta(1 - x - f)E\{V_{n^{s'}}\} \quad (19)$$

This equation corresponds to the marginal value of a worker to the household and enters into the wage bargaining problem described below.

3.2 Solution to the Entrepreneur's Problem

The entrepreneur's problem is complicated by the presence of occasionally binding incentive compatibility constraints (ICC) that result from the inclusion of financial frictions. Given the assumption that investors always find it optimal to incentivize the entrepreneur, the relevant ICC is:

$$p_H(1 - s^i)\theta'_L(k'^i)^\alpha(n^i)^{1-\alpha} \geq As^i \quad (20)$$

where θ'_L denotes the worst possible productivity shock that could occur tomorrow given today's productivity shock (See Section 4.1 for further details regarding θ'_L).

The fact that the incentive compatibility constraint, Equation (20), occasionally binds implies that we must consider two cases: one in which the ICC binds and one in which the ICC is slack. The basic structure of the entrepreneur's problem is identical for both cases. At the start of a period, the entrepreneur has contracted for a given amount of labor that he must hire. He then rents capital to be combined with this labor for use in his project. The entrepreneur also chooses how many vacancies to post today. This choice of vacancies will influence the amount of labor that he must hire tomorrow.

3.3 Binding Incentive Compatibility Constraint

If the incentive compatibility constraint (ICC) is binding, the entrepreneur chooses s^i , k'^i , and v^i in order to maximize equation (13) subject to equations (7), (8), and (20). The

first order conditions for v^i , s^i , and k'^i , and the envelope condition for n^i are given by:

$$\lambda^i G = q\beta E \left\{ \left(\frac{c}{c'} \right) J_{n'^i} \right\} \quad (21)$$

$$\lambda^i p^i - \mu^i \left[p_H \theta'_L (k'^i)^\alpha (n^i)^{1-\alpha} + A \right] = \beta E \left\{ \left(\frac{c}{c'} \right) p_H \theta' (k'^i)^\alpha (n^i)^{1-\alpha} \right\} \quad (22)$$

$$\lambda^i r - \mu^i \alpha p_H (1-s^i) \theta'_L \left(\frac{k'^i}{n^i} \right)^{\alpha-1} = \beta E \left\{ \left(\frac{c}{c'} \right) (1-s^i) p_H \alpha \theta' \left(\frac{k'^i}{n^i} \right)^{\alpha-1} \right\} \quad (23)$$

$$\begin{aligned} J_{n^i} = & - \left(\lambda^i w - \mu^i p_H (1-\alpha) (1-s^i) \theta'_L \left(\frac{k'^i}{n^i} \right)^\alpha \right) + \beta E \left\{ \left(\frac{c}{c'} \right) (1-s^i) p_H (1-\alpha) \theta' \left(\frac{k'^i}{n^i} \right)^\alpha \right\} \\ & + (1-x) \beta E \left\{ \left(\frac{c}{c'} \right) J_{n'^i} \right\} \end{aligned} \quad (24)$$

where λ^i and μ^i denote the Lagrange multipliers on entrepreneur i 's resource financing and incentive compatibility constraints respectively.

Equation (21) is the Euler equation governing the entrepreneur's vacancy posting decision. It equates the current cost of posting a vacancy (G) with the entrepreneur's expected future benefit from posting a vacancy. Equation (22) is the first-order condition on shares with the left-hand side being the net current benefit of issuing the shares and the right-hand side capturing the discounted future cost. Specifically, the first term on the left of (22) is the increased resources received by the entrepreneur as a result of selling an additional share, which are partially offset by the cost imposed by the tightening of the incentive compatibility constraint captured by the second term on the left. The right side of (22) is the cost to the entrepreneur in terms of lost future profits from selling an additional share of his project to outside investors.

Continuing with the first-order conditions, (23) is the first-order condition on capital. The first term on the left-hand side denotes the current rental cost of acquiring this unit of capital. This current cost is partially offset by the benefit to the entrepreneur through the relaxation of incentive constraint due to increased project size, which is captured by the second term on the left. The right-hand side is the entrepreneur's share of the discounted expected return from purchasing one additional unit of capital. Finally, (24) is the envelope condition of the entrepreneur's problem with respect to labor, and provides a measure of the current marginal value of a worker to the entrepreneur. The first two terms on the right-hand side of (24) capture the current cost and expected future benefit of hiring an additional worker. The third term reflects the future asset value of the additional worker, as with probability $(1-x)$ this employee will remain in the next period, allowing the entrepreneur to save on

hiring costs. In particular, the first term is the net hiring cost after accounting for beneficial incentive effects described earlier and the second term is the entrepreneur's share of the discounted expected return from hiring an additional worker. The dependence of equations (21) and (24) on the values of μ^i and λ^i implies that changes in the degree to which the ICC binds will influence the entrepreneur's marginal value of labor as well as the implicit cost they face when posting vacancies.

It is useful to simplify expressions (22)-(24). Specifically, after using (18) and (21) to eliminate expectations, and using equation (20) (binding with equality) to remove A, equations (22)-(24) simplify to:

$$\lambda^i = 1 + \mu^i \left[\frac{p_H \theta'_L (k'^i)^\alpha (n^i)^{1-\alpha}}{p^i s^i} \right] \quad (22b)$$

$$\mu^i = \left(\frac{p^i s^i}{p_H \theta'_L (k'^i)^\alpha (n^i)^{1-\alpha}} \right) \left[\frac{r k'^i - \alpha(1-s^i)p^i}{\alpha(1-s^i)p^i s^i - r k'^i} \right] \quad (23b)$$

$$J_{n^i} = \frac{(1-\alpha)(1-s^i)p^i}{n^i} + \lambda^i \left[\frac{(1-x)G}{q} - w \right] + \mu^i p_H (1-\alpha)(1-s^i) \theta'_L \left(\frac{k'^i}{n^i} \right)^\alpha \quad (24b)$$

Equations (22b) and (23b) provide expressions for λ^i and μ^i respectively. The value of μ^i signifies the degree to which the entrepreneur's incentive compatibility constraint is binding. During a severe economic downturn, the value of μ^i will be large, indicating that the ICC is strongly binding. However, as the economy recovers, the value of μ^i will gradually decline, indicating the relaxation of the ICC. The value of λ^i provides a measure of the scarcity of funds available to the entrepreneur at the start of his project. Notice that the value of λ^i directly depends on μ^i . It is through this channel that the presence of moral hazard impacts the entrepreneur's financing problem. Thus, during times when the entrepreneur's funds are most scarce, the ICC tightens.

3.3.1 Slack Incentive Compatibility Constraint

If the incentive compatibility constraint (ICC) is slack, the entrepreneur solves the same problem as before, except now he ignores equation (20). Solving the slack ICC problem yields the following important equations for the entrepreneur:

$$G = q\beta E \left\{ \left(\frac{c}{c'} \right) J_{n'^i} \right\} \quad (25)$$

$$J_{n^i} = \frac{(1-\alpha)(1-s^i)p^i}{n^i} + \frac{(1-x)G}{q} - w \quad (26)$$

Equations (25) and (26) are identical to their binding ICC counterparts, (21) and (24b), but with μ^i set to 0 and λ^i set to 1. In addition, solving the slack ICC case also provides another simplifying relationship given by:

$$\alpha(1 - s^i)p^i = rk'^i \quad (27)$$

Equation (27) only holds when the ICC is slack, and in fact, it is equivalent to stating that μ^i equals 0 and λ^i equals 1 (See equations (22b) and (23b)).

3.3.2 Wage Bargain

Wages are set through Nash bargaining over the total surplus generated by an additional match. Typically, this surplus is computed by adding the value of an additional worker to the household, V_{n^i} , and the value of an additional worker to the firm, J_{n^s} . However, V_{n^i} is in terms of utils while J_{n^s} is in terms of goods. Therefore, we define the total surplus of an additional match as:

$$TS = V_{n^i} + \tilde{J}_{n^s} \quad (28)$$

where \tilde{J}_{n^s} is defined as $\frac{1}{c}J_{n^s}$.

Using the expression for total surplus presented in equation (28), wages are set by solving:

$$\begin{aligned} & \max_{V_{n^i}, J_{n^s}} (V_{n^i})^b \left(\tilde{J}_{n^s} \right)^{(1-b)} \\ & \text{s.t.} \\ & TS = V_{n^i} + \tilde{J}_{n^s} \end{aligned}$$

where b denotes the worker's bargaining power. Solving this problem yields:

$$(1 - b)V_{n^s}c = bJ_{n^i} \quad (29)$$

Assuming that the ICC binds, equations (19), (24b), and (29) can be used to derive the following reduced wage equation:

$$w = \frac{\eta c + b \left(\frac{(1-\alpha)(1-s)p}{n} + \lambda \Phi G - \eta c + \mu(1 - \alpha)\Delta p(1 - s)\theta'_L \left(\frac{k'}{n} \right)^\alpha \right)}{1 + b(\lambda - 1)} \quad (30)$$

As mentioned earlier, when the ICC is slack, $\lambda = 1$ and $\mu = 0$. Applying this to the above

equation shows that the wage equation reduces to that found in a standard RBC search model:

$$w = \eta c + b \left[\frac{(1 - \alpha)(1 - s)p}{n} + \Phi G - \eta c \right] \quad (31)$$

Therefore, binding incentive constraints are seen to drive a wedge into wages, disrupting the usual dynamics.

3.3.3 Equilibrium and Market Clearing Conditions

Given that all households have identical preferences and face the same shocks, they will choose the same allocations in equilibrium. Therefore, the following equilibrium conditions will hold:

$$s_t^i = s_t, p_t^i = p_t, v_t^i = v_t, n_t^i = n_t^s = n_t, \lambda_t^i = \lambda_t, \mu_t^i = \mu_t, \hat{R}_t^i = \hat{R}_t = y_t = \theta_t k_t^\alpha n_{t-1}^{1-\alpha}.$$

The goods market clearing condition for the model economy is given by:

$$c + k' - (1 - \delta)k + vG = p_H \theta k^\alpha n_{-1}^{1-\alpha} \quad (32)$$

Given this market clearing condition, the model can be summarized by the following equations, (1)-(5), (8), (17), (18), (21), (22b), (23b), (30), (32). These 13 equations contain the 14 distinct endogenous variables, $n, v, m, f, q, \Phi, s, p, w, r, k, c, \lambda$ and μ . In order to close the system, an equation must be added. If the ICC is slack, equation (27) is added, but if the ICC binds, equation (20), the ICC itself, is used to close the system.

4 Calibration and Approximation

In this section we outline the strategy used to calibrate our benchmark model to features of the U.S. data. Since this model contains both financial frictions and labor market search frictions, we provide an overview of how we match the parameters related to these frictions to the data.

4.1 Production and Preference Parameters

We calibrate the model to a quarterly frequency and set the discount factor, β , equal to 0.99, targeting an annual rate of time preference equal to 4%. The depreciation rate, δ , is

set to 0.02, targeting an annual depreciation rate of 8%. The value of α in the production function is set so that capital's share of output is equal to $\frac{1}{3}$.¹⁰ These values, while grounded in observable data, are also supported by existing business cycle literature.

The productivity shock process is given by:

$$\log(\theta_t) = \rho \log(\theta_{t-1}) + \epsilon_t$$

where $\epsilon \sim N(0, \sigma^2)$ with a truncated lower bound of $\epsilon_L = -2.5\sigma$ and a symmetric truncated upper bound of $+2.5\sigma$. The truncation at the lower end is required to compute the worst possible productivity shock that is used in the ICC. The value is computed as:

$$\log(\theta_{L,t+1}) = \rho \log(\theta_t) + \epsilon_L$$

The truncation at the upper end is simply imposed to maintain the symmetry of the shock process. Following Hagedorn and Manovskii (2008), we set the values of ρ and σ to target the first-order autocorrelation and volatility of (expected) average labor productivity (ALP) observed in the data.

4.2 The Labor Market

Calibrating the labor market of our model requires setting values for the following five parameters: the exogenous separation rate, x , the matching function parameter, γ , the disutility of work, η , vacancy posting costs, G , and worker's bargaining power, b . The literature presents several strategies for setting these parameter values. As our main objective is to demonstrate the importance of financial frictions in matching the relative volatility of labor market variables in an RBC search model, we adopt a calibration strategy similar to that presented in Hagedorn and Manovskii (2008). This calibration strategy is known to resolve the Shimer puzzle and allow the standard DMP search model to match the labor market volatility observed in the data. However, Atolia, Gibson, and Marquis (2016b) have demonstrated that this strategy is not sufficient to match labor market volatility in the extended RBC search model when curvature is introduced into the utility function. Therefore, this is the natural calibration choice for introducing financial frictions and assessing their role in bringing labor market volatility in line with the data.

Following standard convention, the exogenous separation rate, x , is set to target a steady

¹⁰The presence of vacancy posting costs prevent us from matching the income shares exactly. However, since posting costs are small, $\alpha = \frac{1}{3}$ delivers a labor share of 0.66.

Table 1: Parameters Values and Empirical Targets

Parameters	Benchmark ^a	No FF, with recal	Empirical Targets	Target Values
α	0.333	0.333	Capital's Share of Output	0.333
β	0.990	0.990	Annual Rate of Time Preference	0.040
δ	0.020	0.020	Annual Depreciation Rate	0.080
ρ	0.933	0.918	Auto Correlation of ALP	0.878
σ	0.010	0.010	Volatility of ALP	2.000
x	0.036	0.036	Unemployment in Steady State	0.055
γ	2.256	2.175	Mean Job Finding Rate	0.592
η	0.380	0.379	Mean Labor Market Tightness	0.720
g	0.448	0.397	Hiring Cost Relative to Output	0.010
b	0.014	0.028	Elasticity of Wages	0.449
A	1.792(0)	0.000	Frequency of Binding ICC	13.600
p_H	0.900	0.900	A Scale Value	Free

^a The Benchmark calibration and “No FF, no recal” have the same parameters, except the agency problem has been shutdown in the latter case by setting $A = 0$.

state unemployment rate of 5.5%. Following Hagedorn and Manovskii (2008) we set workers bargaining power, b , to target the elasticity of wages with respect to average labor productivity observed in the data, 0.449. Also following Hagedorn and Manovskii (2008), we set the the values of the matching function parameter, γ and the disutility of work, η , to target the mean job finding rate and mean value of labor market tightness. However, as our model is calibrated to a quarterly frequency, we target different values. Specifically, we follow Pissarides (2009) and target an average quarterly job finding rate of 0.59 and an average labor market tightness of 0.72. Lastly, we set vacancy posting costs to target a 1% hiring cost to aggregate output ratio (Christiano, Eichenbaum and Trabandt, 2015), yielding a very small value for hiring costs. A full description of parameter values and targets can be found in Table 1.

4.3 The Financial Friction

Following Atolia, Gibson, and Marquis (2016a), we use the asymmetric property of the spread between the rate paid on 3-month non-financial commercial paper and 3-month U.S. Treasury bills to calibrate the level of financial frictions present in the model. Figure 1

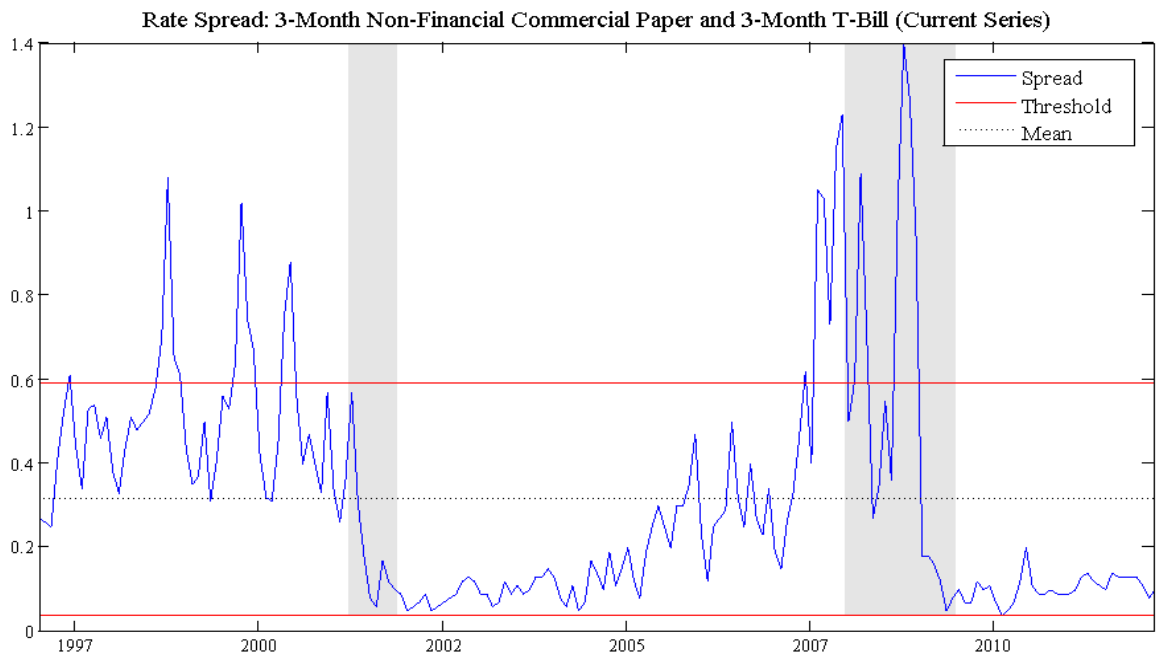
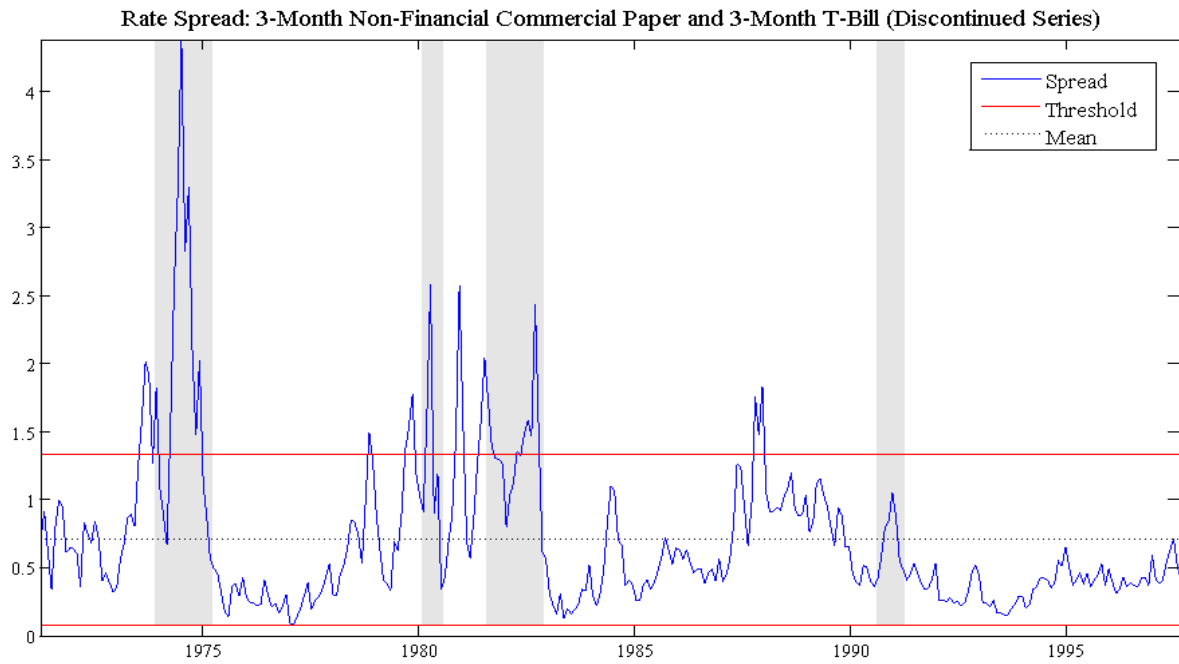


Figure 1: Rate Spread Data

presents plots of this rate spread along with a symmetric band that is formed by reflecting the spread’s minimum about its mean. Inspection of Figure 1 indicates that while this rate spread normally remains within this symmetric band, it occasionally spikes outside of the band and these spikes are concentrated during periods of economic downturn. These large asymmetric swings present in this rate spread are taken “as being indicative of the presence of moral hazard which exposes investors to a greater share of the increase in risk that accompanies downturns” (Atolia, Gibson, and Marquis, 2016a). In our model, the entrepreneur’s shadow price of funds, λ , captures the divergence in value of funds between firms and investors. Given this feature of the model, we set the entrepreneur’s agency rent, $A = p_H \frac{B}{\Delta p}$, so that the simulated time-path for λ spikes outside its symmetric band just over 13% of the time, matching the frequency observed in the rate spread data.¹¹

4.4 Numerical Approximation Methods

Given the presence of an occasionally binding ICC, we are unable to utilize standard linearization techniques that are routinely used throughout the labor search and RBC literatures. As an alternative, we approximate a solution to our model using the Generalized Stochastic Simulation Algorithm (GSSA) proposed by Judd et al (2011) and Maliar and Maliar (2014). GSSA extends the Parameterized Expectations Algorithm (PEA) presented in den Haan and Marcet (1990), which is known to work well for models with occasionally binding constraints (See Christiano and Fisher, 2000). At their core, PEA and GSSA share the same basic structure; a repeated application of simulation and estimation. Instead of recovering the model’s policy rules, PEA and GSSA approximate the model’s conditional expectation function using a simple function of the model’s state variables. Once the model’s expectation equations have been removed, the model can be simulated to recover time series for each endogenous variable. This data is then used to update the parameter values of the approximating function. This process of simulation and estimation is repeated until the change in the approximating functions’ parameter values is sufficiently small.

While PEA and GSSA share the same basic structure, GSSA improves upon PEA in several ways.¹² First, while PEA approximates the conditional expectation function using a product of monomials in the model’s state variables, GSSA uses a more flexible orthogonal

¹¹The extension of the model in Appendix A allows us to match not only the frequency of spikes in the rate spread but also the amplitude of those spikes.

¹²While we provide a basic overview of the improvements of GSSA over PEA, the interested reader is referred to Maliar and Maliar (2014) for further details.

polynomial.¹³ In particular, we use a fourth-order Hermite polynomial in our model’s state variables to approximate our conditional expectation equations. Second, PEA uses nonlinear least squares (least square learning) in the regression step to update the parameters of the approximating function. This regression is prone to ill-condition problems and may lead to numeric instability of the approximating parameters. To deal with this issue, we follow Judd et al (2011) and Maliar and Maliar (2014) and replace nonlinear least squares with a singular value decomposition (SVD) regression which avoids the inversion of a potential unstable matrix. Lastly, while PEA uses a variable’s realized future value as its current period expectation, GSSA uses deterministic integration techniques to accurately evaluate the model’s conditional expectation function. In our model, we use 9-node Gauss-Hermite quadrature to accurately evaluate our expectations in the regression step. In total, these improvements allow GSSA to recover a reliable, stable, and accurate approximation of our model using a 10,000 period simulation path, which is substantially shorter than the path that would be required with PEA.

5 Results

We now present the results showing the performance of our model with financial frictions in matching the labor market moments. We argue that our benchmark model performs well in matching these moments and allows one to overcome the Shimer puzzle in an RBC search environment. We also clarify the role of financial frictions, in particular their *interaction* with labor market search frictions in delivering these results. While understanding the role played by financial frictions in amplifying labor market volatility in an RBC search model is the primary focus of the paper, we also find that the introduction of financial frictions improves the *dynamic* response of macro and labor market variables to productivity shocks. Specifically, unemployment now displays a distinct hump-shaped response as suggested by the data (Fujita and Ramey, 2007 and Petrosky-Nadeau, 2014).

¹³The primary issue with the approximating function chosen in PEA is that higher order terms tend to be highly correlated with lower order terms. Therefore, while in theory it should be possible to increase PEA’s approximation accuracy to an arbitrary level through the inclusion of higher order terms, multicollinearity induced by correlated regressors makes this a practical impossibility. By using orthogonal polynomials, GSSA avoids this issue.

Table 2: Summary of Second Moments

	Data	Benchmark	No FF, with recal	No FF, no recal
Calibration Targets ^a				
Wage Elasticity	0.45	0.44	0.44	0.44
Mean Job Finding Rate	0.59	0.59	0.60	0.63
Mean Labor Market Tightness	0.72	0.73	0.73	0.77
Volatility of ALP	2.00	2.00	2.00	2.17
Auto Correlation of ALP	0.88	0.87	0.88	0.89
Binding Rate	0.14	0.14	0.00	0.00
Hiring Cost Relative to Output	0.01	0.01	0.01	0.01
Labor Market Second Moments ^b				
Volatility of Vacancies	20.20	22.49	9.48	10.09
Volatility of Unemployment	19.00	21.43	8.96	9.65
Volatility of Labor Market Tightness	38.20	35.55	15.17	16.47
Financial Multipliers (ratios of Benchmark-to-No FF volatilities)				
Vacancies			2.37	2.23
Unemployment			2.39	2.22
Labor Market Tightness			2.34	2.16
Traditional Business Cycle Moments ^c				
Volatility of Output	1.57	1.65	1.36	1.43
Volatility of Consumption	1.29	0.71	0.58	0.64
Volatility of Investment	6.61	12.77	7.86	7.88
Cyclicalilty of Consumption	0.85	0.91	0.94	0.95
Cyclicalilty of Investment	0.88	0.81	0.98	0.97

^a The “No FF, no recal” case is not recalibrated to match targets. Instead, the same parameter values found for the “Benchmark” calibration are used and the effect of moral hazard is removed by setting $A = 0$.

^b The data values reported in this section come from Shimer (2005). Volatilities are reported as percent standard deviations.

^c Data on Y , c , and inv comes from the National Income and Product Accounts tables and covers the period of 1951-2003. The time period is chosen to retain comparability with Shimer (2005). Specifically, Y is computed as GDP less consumer durables, c is computed as non-durables plus services, and inv is computed as Gross Private Domestic Investment.

5.1 Labor Market Performance

We start by investigating the performance of our benchmark model with financial frictions in generating realistic levels of labor market volatility. Inspection of the first two columns of Table 2 indicate that when we match our empirical targets, our benchmark model generates labor market variables that are as volatile as the data. For example, the model predicts the volatility of vacancies, unemployment, and labor market tightness expressed as percent standard deviations to be 22.49, 21.43, and 35.55, while the data suggest that these values are 20.20, 19.00, and 38.20 respectively.¹⁴ It should be noted that HM also match the high volatility of the labor market variables and as we adopted a calibration strategy closely related to theirs, it is possible that the calibration, not the presence of financial frictions, is behind our result. However, HM demonstrate their results within the standard DMP model with linear utility (risk neutral agents and infinite Frisch elasticity) and linear production (in labor with no capital). Atolia, Gibson, and Marquis (2016b) demonstrate that the HM calibration fails to generate sufficient labor market volatility within an RBC search model with curvature in utility and where capital is included as an additional factor of production. Therefore, it is clear that further investigation is needed.

Table 3 presents the correlations between unemployment, vacancies, labor market tightness, and the job finding rate. Inspection of Table 3 indicates that the benchmark model also replicates the data along this dimension reasonably well. The model correctly predicts the direction of correlation between all of these variables, and is mostly accurate on the degree of correlation. The one short-coming is the degree of negative correlation between unemployment and vacancies. The model estimates this to be approximately -0.31, while the data suggests that the value should be closer to -0.89. Thus our benchmark model provides a fit to the data on the correlations between labor market variables similar to that reported in Shimer (2005) and Hagedorn and Manovskii (2008), excepting our under-estimated correlation between unemployment and vacancies.

5.2 The Role of Financial Frictions

In this section, we assess the role played by financial frictions in driving the previously mentioned results. We start by comparing the performance of the benchmark model with the variant of the model labeled “No FF, with recal”, where we remove the effect of moral hazard by setting $A = 0$, thereby eliminating the borrower’s private benefit and hence the agency

¹⁴The ability of the model to generate volatility of labor market variables is robust to the inclusion of internal finance in the model as shown in Appendix A.

Table 3: Labor Market Correlations

Variable	Model	Unemployment	Vacancies	Tightness	Job Finding Rate
U	Data ^a	1.00	-0.89	-0.97	-0.95
	Benchmark	1.00	-0.31	-0.80	-0.77
	No FF, with recal	1.00	-0.35	-0.81	-0.81
	No FF, no recal	1.00	-0.39	-0.83	-0.82
v	Data	-	1.00	0.980	0.90
	Benchmark	-	1.00	0.82	0.83
	No FF, with recal	-	1.00	0.83	0.83
	No FF, no recal	-	1.00	0.84	0.84
Φ	Data	-	-	1.00	0.95
	Benchmark	-	-	1.00	0.99
	No FF, with recal	-	-	1.00	1.00
	No FF, no recal	-	-	1.00	1.00
f	Data	-	-	-	1.00
	Benchmark	-	-	-	1.00
	No FF, with recal	-	-	-	1.00
	No FF, no recal	-	-	-	1.00

^a The “No FF, no recal” case is not recalibrated to match targets. Instead, the same parameter values found for the “Benchmark” calibration are used and the effect of moral hazard is removed by setting $A = 0$. The data values reported in this Table come from Shimer (2005).

cost in lending, and recalibrate the remaining parameters so that the model still hits all other empirical targets described above. The results from this variant are reported in Tables 2 and 3 alongside that for the benchmark model. Table 3 shows that both models deliver similar performance in terms of the correlation structure of the labor market variables. However, as Table 2 shows the model labeled “No FF, with recal” fails to deliver on the volatility of the labor market variables. In particular, we report financial multipliers, defined as the ratio of standard deviations of labor market variables in the benchmark versus the “No FF, with recal” case, indicating that the financial frictions more than double the volatility

of vacancies, unemployment, and labor market tightness.¹⁵ It is, therefore, clear that the presence of financial frictions, is necessary to bring the model’s results in line with the U.S. data on the relative volatility of labor market variables.

In order to provide insight into the manner in which financial frictions help the model match the data, we solve another variant labeled “No FF, no recal” in which we shut down financial frictions in the benchmark model (again by setting $A = 0$), but do not recalibrate other model parameters. Comparing the results of this variant with the benchmark case shows that the inclusion of financial frictions increases volatility of labor market variables by almost 100% (or more), but it causes output volatility to increase by only approximately 16%. The fact that financial frictions add more to the volatility of labor market variables than output is very significant. It highlights the quantitative importance of the *interactions* between financial frictions and labor market search frictions in driving the results of the model and deserves further discussion.

5.2.1 Interaction of Financial and Labor Market Frictions

To understand the mechanism underlying the interaction of financial frictions and labor market search frictions, we need to first describe how financial frictions affect the economy. The additional volatility found in the benchmark model arises due to the binding of the entrepreneurs’ incentive compatibility constraints during times of sufficiently low aggregate economic activity. During times of economic prosperity, the expected revenue of a project is high enough to naturally incentivize the entrepreneur to work diligently. During a sufficiently severe recession, the expected revenue of a project falls, and it becomes increasingly beneficial for the entrepreneur to shirk and engage in a privately beneficial activity. The investor incentivizes the entrepreneurs in order to prevent this shirking by purchasing fewer shares in the project, thereby leaving a larger piece of the project’s profits for the entrepreneur. While this act of purchasing fewer shares leads to diligent behavior on the part of the entrepreneurs, it also reduces the funds available at the start of the projects, thereby reducing project size and revenue above and beyond that caused by the reduction in productivity.

In order to see the source of interaction between financial and labor market frictions, note that during a typical economic downturn productivity falls, leading to a reduction in the entrepreneur’s marginal value of a worker. This reduction in the worker’s value reduces the entrepreneur’s incentive to post vacancies and attract additional workers. As a consequence, unemployment increases. If the downturn is severe enough to activate the

¹⁵We thank a referee for this suggestion.

financial friction, then the accompanying shortage of funds for the projects as mentioned above causes the shadow price of funds to rise. Since the entrepreneur must use these funds to post vacancies, this increase in the shadow price increases the implicit cost of posting vacancies faced by the entrepreneur. This increase in the cost of posting vacancies further reduces the entrepreneur's incentive to post vacancies, leading to an even greater increase in unemployment.

The adverse effects of financial frictions on the benchmark model economy, as described above, can be seen in Figures 2 and 3 which present simulated time paths for output, consumption, unemployment, and labor market tightness, as well as other key variables of the model. Each sub-plot contains the time path for all three variants of the model. The economies diverge when financial frictions are operational. For example, consider the time interval between periods 50 and 125. During this time, aggregate productivity is sufficiently low so as to cause the incentive compatibility constraint to bind in the benchmark model with financial frictions. The binding of the ICC is reflected by the spikes in the shadow price of funds and the drop in shares during this time. This additional reduction in shares causes output, consumption, and capital to all fall in the financial-friction ridden economy relative to those where the financial frictions have been removed. The presence of financial frictions can also be seen to affect the labor market variables in a similar way. For example, unemployment spikes during this time, while the job finding rate falls much farther than it would if financial frictions were not present.

Not surprisingly, as lending dries up during severe recessions, the recovery period can be significantly drawn out. To illustrate the importance of credit constraints during recessionary periods, the model is subjected to a -2.5 standard deviation shock to productivity in the first two periods, then allowed to recover thereafter. Plots of the impulse response functions for productivity, output, and the key labor market variables: unemployment, labor market tightness, and the job-finding rate, are presented in Figures 4 and 5. While these figures contain all three variants of the model, we will focus on the benchmark and No FF, no recal cases as they illustrate the difference between a model with and without financial frictions under the same shock process.¹⁶ The graphs demonstrate how the labor market is much more sharply impacted by the adverse productivity shocks in the presence of agency costs arising from moral hazard. The interactions between financial and labor market frictions in the benchmark model strengthens the endogenous propagation of shocks to the macro variables. In particular, output, unemployment, and wages show prominent hump-shaped

¹⁶Also, the results for the No FF, with recal case are very similar to those for the No FF, no recal case.

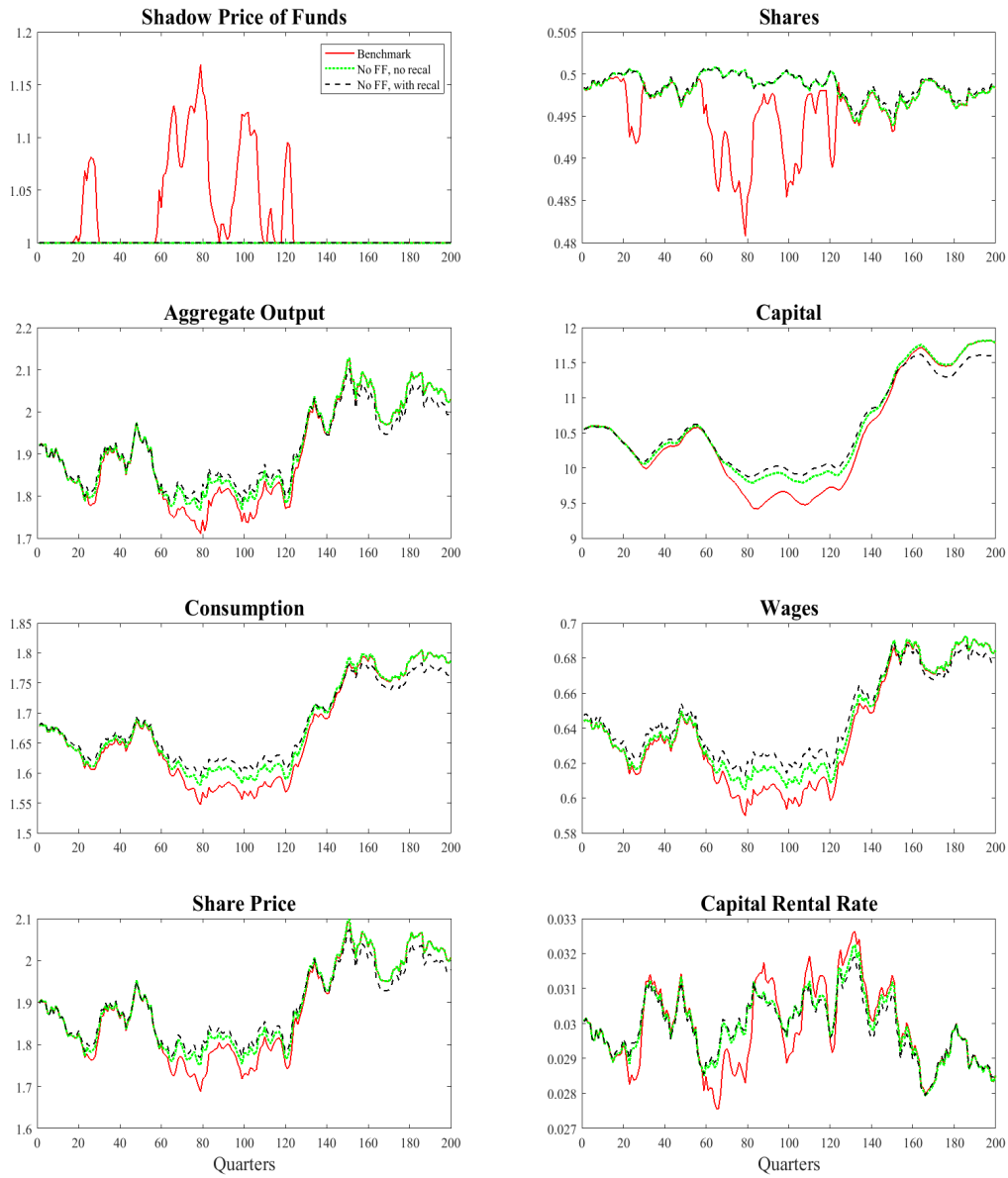


Figure 2: Simulated Time Paths I

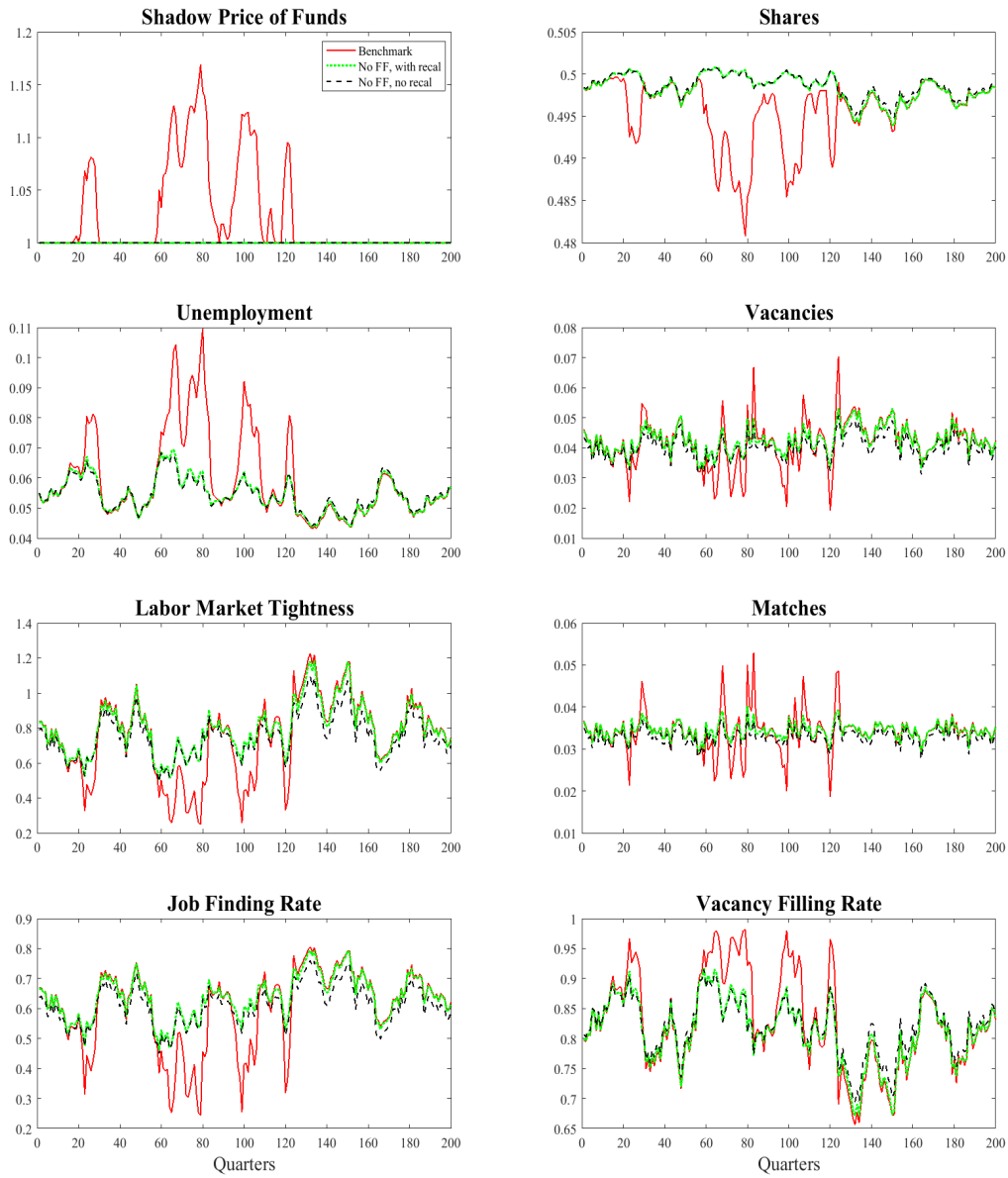


Figure 3: Simulated Time Paths II

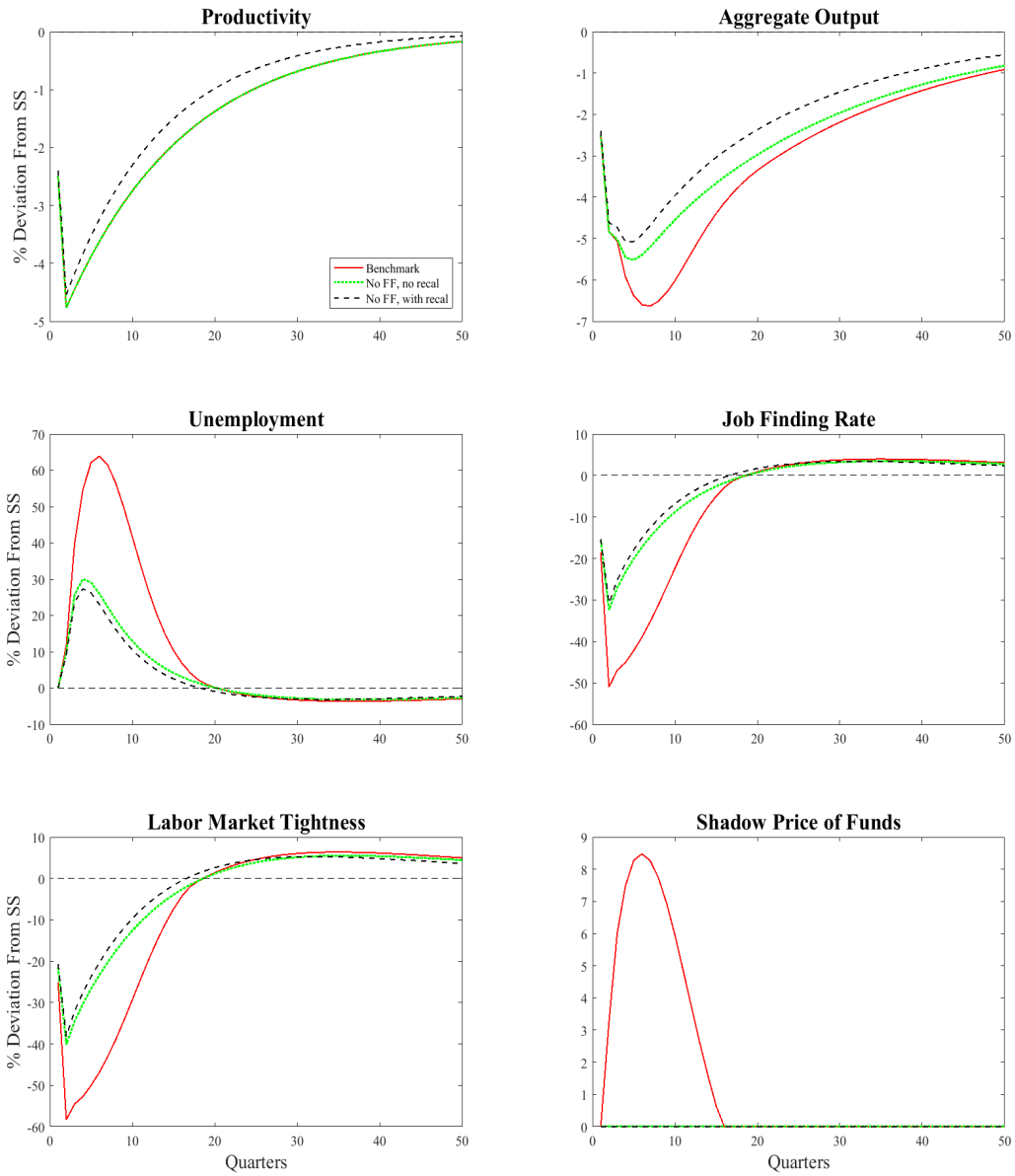


Figure 4: Impulse Response Functions I

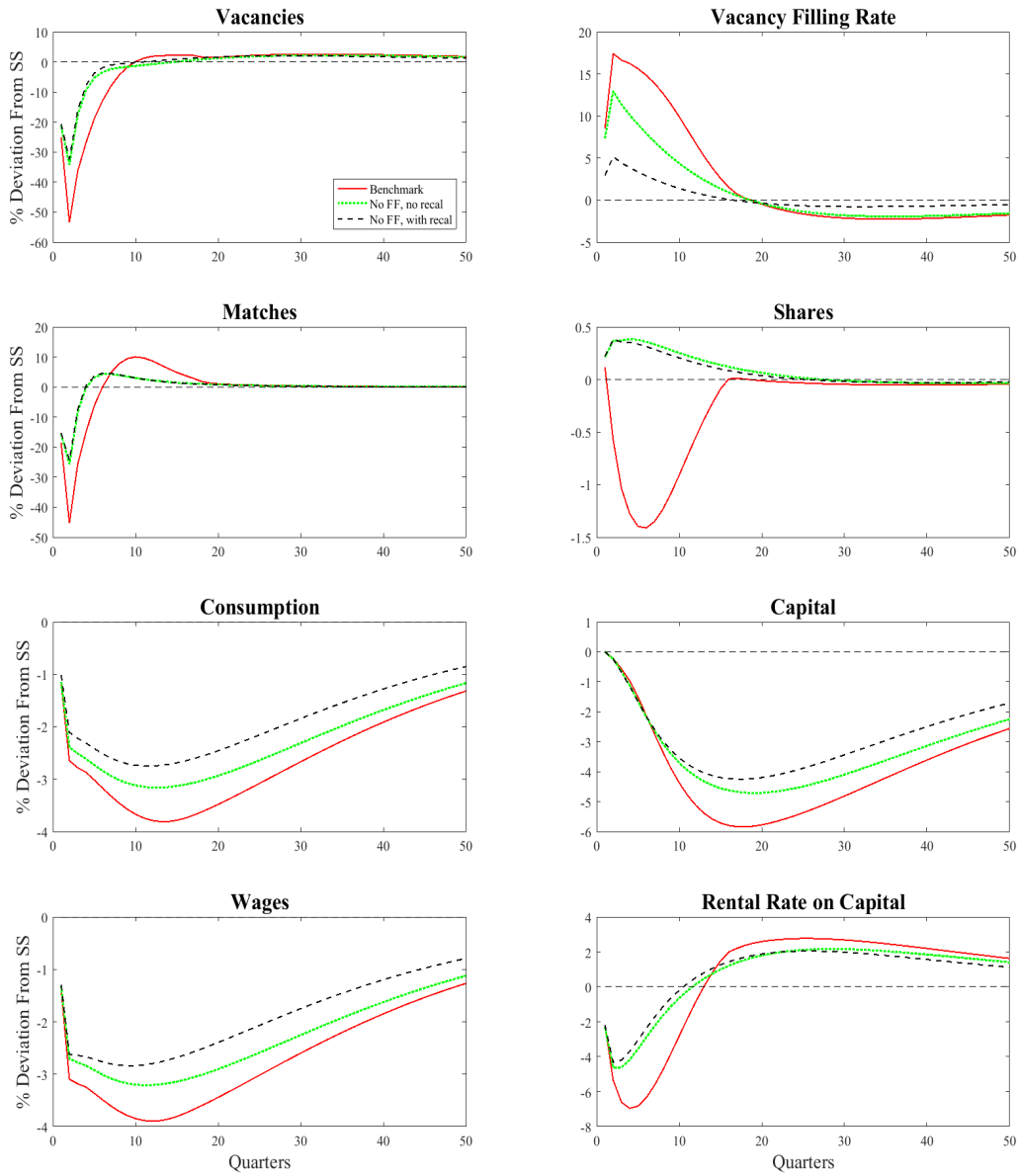


Figure 5: Impulse Response Functions II

responses for the benchmark model that peak two quarters later than the response found for the No FF, with recal case. For many labor market variables other than unemployment, such as the job-finding rate, labor market tightness, and the vacancy-filling rate, while the peak response occurs in the same period in both models, there is slow recovery in the sense that the return to the pre-shock level occurs at a slower pace for a number of periods after the peak response.

6 Conclusion

This paper develops a model in which financial frictions arising from firm-level moral hazard interact with labor market search frictions. The results suggest that the interaction of these frictions is vital for the model to successfully resolve the Shimer puzzle. The financial frictions were shown to be necessary for the model to reproduce the high observed relative (with respect to average labor productivity and output) volatility of the labor market variables. A variant of the model without financial frictions, while successful in replicating the observed volatility of output and average labor productivity, significantly underestimated the volatility of vacancies, unemployment, and labor market tightness. Also, when our financial frictions are active, they are shown to generate internal shock propagation, which causes the peak response of output, unemployment, and wages to be delayed. Many other labor market variables also show slower recovery—the return to pre-shock level occurs at a slower pace for a number of periods after the peak response.

There are a number of potentially promising extensions that readily suggest themselves. For example, firms of different sizes use different forms of finance and face differential tightening of credit constraints over the business cycle. It may be possible to extend the model to include heterogeneous firms whose access to finance varies depending on their current size. This may also have additional cross-sectional implications for unemployment over the business cycle.

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A Appendix: Self-Finance and the Amplitude of Credit Spikes

In this appendix we extend the model developed in the body of the paper to allow for pre-determined internal finance. Entrepreneurs are now required to fund some of their project's costs by bringing funds from their own household. This extension, in addition to providing the added realism of both internal and external finance, also provides an extra free parameter which can be used to target the magnitude of financial shocks present in our model economy. Therefore, while the baseline model in the paper is calibrated to match the frequency of large spikes in credit spreads, the extended model presented here is capable of matching both the frequency and amplitude of these episodes.

A.1 Model Extension: Predetermined Self-Finance

In the extended model, entrepreneurs still sell shares in their projects to outside investors, but they now also raise internal finance by issuing a predetermined number of shares (z) to their own household in exchange for internal finance. Modifying the model in this way results in only two equations being altered. First, the entrepreneur's budget constraint (equation (8) in main text) becomes:

$$wn + rk' + vG \leq p[s + z] \quad (\text{A.8})$$

Next, the household's budget constraint (equation (14) in main text) must be adjusted to accommodate the additional expenditure by households on provision of internal finance.

$$c + \int_0^1 p^j s^j dj + z \int_0^1 p^i di + k' - (1 - \delta)k \leq wn + \int_0^1 \pi^i di + rk' + \int_0^1 p_H \hat{R}^j s_{-1}^j dj \quad (\text{A.14})$$

Besides these two changes, the rest of the model, including the structure of the labor market and objective functions faced by entrepreneurs and households, remains unchanged.

A.2 Calibration: Amplitude of Credit Spikes

As seen above, the extended model with predetermined self-finance introduces a single new parameter, z , which governs the amount of finance that must come from within the entrepreneur's own household. The calibration strategy presented in the body of our paper is retained to pin down all overlapping parameters. In addition, we set z to target the magnitude of spikes observed in U.S. rate spread data; specifically, the peak spread between three-month non-financial commercial paper and U.S. treasuries, which is approximately 4.5% (see Figure 1). As Figure A.1 shows, we are able to generate spikes of similar magnitude in the shadow price of funds, λ , by setting $z = 0.5$, implying about 67% internal finance.¹⁷ Figure A.1 also plots λ for the baseline model presented in the body of the pa-

¹⁷We are actually generating a slightly lower peak of 4.38% in the first 150 quarters of our simulation that are used for our comparison.

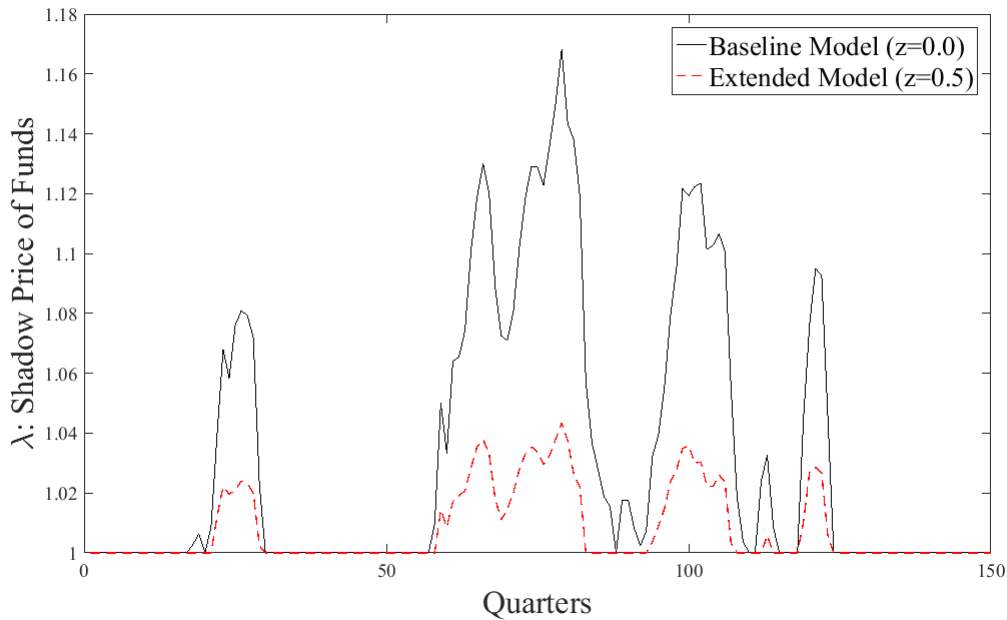


Figure A.1: Comparison of Shadow Price Paths

per and clearly shows that the magnitude of the response in λ diminishes greatly with the increase in z , with over a four-fold reduction in amplitude being observed in the figure.

While the results presented in Figure A.1 clearly show that z can be adjusted to target the amplitude of spikes in λ that result due to our financial friction, we must still show that the primary results of our model are robust to this extension. Specifically, are we still able to generate sufficient labor market volatility when the size of financial shocks have been reduced to an empirically relevant level (i.e., when both the frequency and amplitude of the credit spreads are considered as targets)? Table A.1 presents the calibration targets and labor market volatilities for both our baseline model ($z = 0.0$) and the extended model ($z = 0.5$). Inspection of this table clearly shows that once both model specifications are calibrated to match our common empirical targets, we are able to generate similar levels of labor market volatility that align closely with that observed in the data. For example, the baseline and extended models generate volatility measures for labor market tightness of 35.55 and 34.94, which are close to the value of 38.20 observed in the data. Therefore, our results indicate that the asymmetric properties of price spikes (and more generally, our credit friction), not their amplitude, is crucial for our observed results.

Table A.1: Predetermined Self-Finance

	Data	Baseline Model ^a ($z=0.0$)	Extended Model ^b ($z=0.5$)
Calibration Targets			
Wage Elasticity	0.45	0.44	0.44
Mean Job Finding Rate	0.59	0.59	0.59
Mean Labor Market Tightness	0.72	0.73	0.72
Volatility of ALP	2.00	2.00	2.00
Auto Correlation of ALP	0.88	0.87	0.87
Binding Rate	0.14	0.14	0.14
Hiring Cost Relative to Output	0.01	0.01	0.01
Labor Market Second Moments			
Volatility of Vacancies	20.20	22.49	22.65
Volatility of Unemployment	19.00	21.43	20.91
Volatility of Labor Market Tightness	38.20	35.55	34.94

^a The baseline specification with $z = 0.0$ corresponds to the baseline model from the paper.

^b The parameters of the alternative specification with $z = 0.5$ have been recalibrated to ensure that this specification maintains all empirical targets suggested by our calibration strategy.